MA 526 **Name:**________________________

Taken-Home Test October 24, 2007.

1. Derive a formula (similar to the *d'Alembert's formula*) for the solution $v(x,t)$ of the Cauchy problem for the one-dimensional Klein-Gordon equation:

$$\begin{cases} v_{tt} = c^2 v_{xx} - m^2 c^2 v & \text{for } x \in \mathbb{R} \text{ and } t > 0, \\ v(x,0) = g(x), \qquad v_t(x,0) = h(x) & \text{for } x \in \mathbb{R}. \end{cases}$$

2. Use the Poisson's method of *spherical means* to find a formula for the solution $v(x,t)$ of the Cauchy problem for the three-dimensional Klein-Gordon equation:

$$\begin{cases} v_{tt} = c^2 \triangle v - m^2 c^2 v & \text{for } x \in \mathbb{R}^3 \text{ and } t > 0, \\ v(x,0) = g(x), \qquad v_t(x,0) = h(x) & \text{for } x \in \mathbb{R}^3. \end{cases}$$

3. Find the solution $u(x,y,z,t)$ of initial value problem

$$\begin{cases} u_{tt} = u_{xx} + u_{yy} + u_{zz} \\ u(x,y,z,0) = x^2 + y^2, \qquad u_t(x,y,z,0) = 0. \end{cases}$$

4. Consider a flexible beam with clamped ends at $x = 0$ and $x = 1$. Small wave motion $u(x,t)$ in the beam satisfies

$$\begin{cases} u_{tt} + \gamma^2 u_{xxxx} = 0 & \text{for } 0 < x < 1 \text{ and } t > 0 \\ u(0,t) = 0 = u(1,t) & \text{for } t \geq 0 \\ u_x(0,t) = 0 = u_x(1,t) & \text{for } t \geq 0, \end{cases}$$

where γ^2 is a constant depending on the shape and material of the beam. Show that the energy $E(t) = \int_0^1 (u_t^2 + \gamma^2 u_{xx}^2)\, dx$ is conserved.

5. Use the energy method to establish the uniqueness of the solution $u(x,t)$ of the initial-boundary value problem

$$\begin{cases} u_{tt} + c^2 u_{xx} = 0 & \text{for } 0 < x < L \text{ and } t > 0, \\ u(x,0) = g(x), \quad u_t(x,0) = h(x) & \text{for } 0 < x < L \\ \alpha u(0,t) + \beta u_x(0,t) = 0, \quad \gamma u(L,t) + \delta u_x(L,t) = 0 & \text{for } t \geq 0, \end{cases}$$

where α, β, γ, δ, L are constants with $\alpha^2 + \beta^2 \neq 0$, $\gamma^2 + \delta^2 \neq 0$, and $L > 0$.