Math 508 **Solutions to Quiz #1** **(6/4/03)**

1. Use Gaussian or Gauss-Jordan eliminations to find all the solutions of

$$\begin{cases} x_1 + x_2 + 0x_3 + x_4 = 7 \\ x_1 + 0x_2 + x_3 + x_4 = 4 \\ 0x_1 + x_2 + x_3 + x_4 = 10 \end{cases}$$

Solution. We first apply a series of row operations for the augmented matrix untill the coefficient matrix A becomes its reduced row echelon form (which is unique):

$$[A|b] = \begin{bmatrix} 1 & 1 & 0 & 1 & | & 7 \\ 1 & 0 & 1 & 1 & | & 4 \\ 0 & 1 & 1 & 1 & | & 10 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 & 1 & | & 7 \\ 0 & -1 & 1 & 0 & | & -3 \\ 0 & 1 & 1 & 1 & | & 10 \end{bmatrix}$$
$$\to \begin{bmatrix} 1 & 0 & 1 & 1 & | & 4 \\ 0 & -1 & 1 & 0 & | & -3 \\ 0 & 0 & 2 & 1 & | & 7 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & 1 & | & 4 \\ 0 & 1 & -1 & 0 & | & 3 \\ 0 & 0 & 1 & 1/2 & | & 7/2 \end{bmatrix}$$
$$\to \begin{bmatrix} 1 & 0 & 0 & 1/2 & | & 1/2 \\ 0 & 1 & 0 & 1/2 & | & 13/2 \\ 0 & 0 & 1 & 1/2 & | & 7/2 \end{bmatrix}.$$

Hence, x_4 is a free variable and the general solution of the system is:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 13/2 \\ 7/2 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} -1/2 \\ -1/2 \\ -1/2 \\ 1 \end{bmatrix},$$

where α is a real parameter.

2. Find the determinant of A by any row or column expansion, where

$$A = \begin{bmatrix} 1 & 2 & 0 & 50 \\ -2 & 1 & -100 & 5 \\ 1 & 3 & 0 & 0 \\ 2 & 1 & 0 & 0 \end{bmatrix}.$$

Solution. Since third column contains only one nonzero entry, we expand along this column to get

$$detA = -100(-1)^{2+3} det \begin{bmatrix} 1 & 2 & 50 \\ 1 & 3 & 0 \\ 2 & 1 & 0 \end{bmatrix} = 100 \cdot det \begin{bmatrix} 1 & 2 & 50 \\ 1 & 3 & 0 \\ 2 & 1 & 0 \end{bmatrix}.$$

We then expand along the third column for the above determinant to get

$$detA = 100 \cdot 50(-1)^{1+3} det \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} = 5000(1-6) = -25000.$$

Below are the facts related to a matrix in row echelon form or reduced row echelon form.

Definition. A matrix is said to be in row echelon form if the following conditions are satisfied:

- Any row containing a nonzero entry precedes any row in which all the entries are zero (if any).
- The first nonzero entry in each row is called the leading entry of its row and it occurs in a column to the right of the leading entry in any preceding row.
- If a column has a leading entry, then all entries below the leading entry in that column are zero.

Theorem. Gaussian elimination (a series of forward row operations) transforms any matrix into a row echelon form.

Definition. A matrix is said to be in reduced row echelon form if the following conditions are satisfied:

- It is in row echelon form.
- All the leading entries are 1.
- The leading entry 1 is the only nonzero entry in its column.

Theorem. Gauss-Jordan elimination (a combination of forward and backward row operations) transforms any matrix into its reduced row echelon form, which is unique.