MA 508 **Name:**________________________

Final Test May 2, 2006.

1. Decide if $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$ are linearly independent and then find a basis for the subspace spanned by these four vectors.

2. Let W be the subspace of $\mathbb{R}^3$ that consists of all the vectors in the plane $x_1 + x_2 - x_3 = 0$.

(i) Find the orthogonal projection matrix from $\mathbb{R}^3$ to W.

(ii) Find the vector in W that is closest to the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

3. (i) Verify that $A = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$ is an orthogonal matrix.

(ii) Find A^{-1}.

(iii) Let λ be eigenvalue of A. Show that $|\lambda| = 1$.

4. Let $W = span\{v_1, v_2\}$ where $v_1 = (1, 1, -1)^t$ and $v_2 = (1, 0, 1)^t$.

(i) Find a vector u with length 1 that is orthogonal to every vector in W.

(ii) Find a Householder matrix H that reflect $\mathbb{R}^3$ through W parallel to u.

(iii) Find Hv_1, Hv_2 and Hu.

(iv) (**Bonus**) Unitarily diagonalize H.

5. Let $W := \{A \in \mathbb{R}^{3\times 3} : A = -A^t\}$.

(i) Show that if $A \in W$, then the entries in the main diagonal of A are zero.

(ii) Show that W is a subspace of $\mathbb{R}^{3\times 3}$, and then find a basis of W.

(iii) Let $A \in W$. Show that A is normal.

(iv) For any real number α and $A \in W$, show that $\alpha I + A$ is normal.

(v) Is the matrix $B = \begin{bmatrix} 2 & 3 & 4 \\ -3 & 2 & 5 \\ -4 & -5 & 2 \end{bmatrix}$ unitarily diagonalizable? Explain your answer.

6. (i) Find a Schur decomposition for $A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$, and then find A^{11}.

(ii) Diagonalize A and then compute A^{11}.

7. Use eigenvalues and eigenvectors to unitarily diagonalize $A = \begin{bmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{bmatrix}$.