MA456/506 (UAH) **Name:**_____

Test 1 March 5, 2008

1. Solve $u_t + u_x + u = e^{x+2t}$ with $u(x,0) = 0$.

2. Consider the Laplace equation on the square D: $0 < x < 1$, $0 < y < 1$ with the boundary conditions $u = 0$ on the top and bottom and the side $x = 1$, and $u(0,y) = y(1-y)$ for $0 \le y \le 1$.

(i) Find the maximum and minimum values of u on the closed square $0 \le x \le 1$, $0 \le y \le 1$? Where do they occur?

(ii) Find an approximate value for $u(1/2, 1/2)$.

3. Find the general solution of the equation

$$u_{xx} - u_{tt} + u_x - u_t = 0.$$

4. Solve $u_{tt} = 4u_{xx}$ for $0 < x < \infty$, $u(0,t) = 0$, $u(x,0) = 1$, $u_t(x,0) = 0$. This solution has singularity; find its location.

5. Use the following steps to solve

$$\begin{aligned} u_{tt} - c^2 u_{xx} &= x^2 \qquad \text{for } t > 0 \text{ and all } x \\ u = x, \quad u_t &= 0 \qquad \text{for } t = 0 \end{aligned}$$

(i) Find a particular solution of the form $u = u_p(x)$ for the equation $u_{tt} - c^2 u_{xx} = x^2$;

(ii) Let $u_h(x,t)$ be the general solution of $u_{tt} - c^2 u_{xx} = 0$. The general solution of $u_{tt} - c^2 u_{xx} = x^2$ is given by

$$u(x,t) = u_h(x,t) + u_p(x).$$

Use the initial data to determine the arbitrary functions in this general solution.

6. Verify that the function

$$u(x,t) = x t^{-3/2} e^{-x^2/4t} \qquad -\infty < x < \infty, \quad t > 0$$

satisfies the heat equation $u_t = u_{xx}$. What is $\lim_{t \to 0^+} u(x,t)$ for each $x \in (-\infty, \infty)$?