

MA 238: TEST 2 (11/10/04)

INSTRUCTIONS: Write your name on the answer sheet and show all the necessary work for each problem.

1. (i) Find general solutions for the equations:

$$(a) \quad y'' + 5y' + 4y = 0, \quad (b) \quad y'' - 6y' + 13y = 0, \quad (c) \quad y'' + 5y = 0.$$

- (ii) Find the solution of the IVP:

$$y'' + 4y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 1$$

.

2. Find a particular solution of $4y'' + 4y' + y = 2xe^{-x}$.

3. Give the equation $y'' + y = \sin x$.

- (i) Find a particular solution y_p for this equation.
(ii) Find a general solution for this equation.

4. Set up the appropriate form for y_p without determining the values of the coefficients.

$$(1) \quad y'' - 2y' + 2y = e^{2x}x \cos x \quad (2) \quad y'' - 2y' + 2y = xe^x \cos x \\ (3) \quad y'' - 2y' + y = xe^x$$

5. Use the method of variation of parameters to find a particular solution to

$$y'' - 4y = e^x + e^{-2x}.$$

6. **[Bonus]** (i) Verify that $y_1 = x$ and $y_2 = x^2$ are two linearly independent solutions of

$$y'' - \frac{2}{x}y' + \frac{2}{x^2}y = 0$$

on $(0, \infty)$.

- (ii) Use the method of variation of parameter to find a particular solution of

$$y'' - \frac{2}{x}y' + \frac{2}{x^2}y = x^{5/2}$$

on $(0, \infty)$.

Answers:

1. (i) (a) $c_1 e^{-4x} + c_2 e^{-x}$; (b) $c_1 e^{3x} \cos 2x + c_2 e^{3x} \sin 2x$; (c) $c_1 \cos \sqrt{5}x + c_2 \sin \sqrt{5}x$.
(ii) $(1 + 3x)e^{-2x}$.

2. $(2x + 8)e^{-2x}$.

3. (i) $-\frac{1}{2}x \cos x$; (ii) $c_1 \cos x + c_2 \sin x - \frac{1}{2}x \cos x$.

4. (1) $[(A_0x + A_1) \cos x + (B_0x + B_1) \sin x]e^{2x}$; (2) $x[(A_0x + A_1) \cos x + (B_0x + B_1) \sin x]e^{2x}$; (3) $x^2(Ax + B)e^x$.

5. $-(\frac{1}{3}e^x + \frac{1}{4}xe^{-2x} + \frac{1}{16}e^{-2x})$.

6. (ii) $\frac{4}{35}x^{9/2}$.