

## MA 238-04: TEST 1 (2/13/07)

1 [20 pts] Find the solution  $y(x)$  of the initial value problem (IVP):

$$\frac{1}{3}y' + y = x^2e^{-3x}, \quad y(0) = 1.$$

2 [20 pts] Find the solution to

$$xy^3y' = x^4 + y^4, \quad y(1) = 1.$$

3 [20 pts] Find the general solutions to

$$y' + y = e^xy^2.$$

4 [20 pts] Use Euler's method with step size  $h = \frac{1}{2}$  to find an approximate value of  $y(2)$ , where  $y$  is the solution of the IVP:

$$y' = y - 2x - 1, \quad y(0) = 1.$$

5 [10 pts] Birth and death rates of animal populations typically are not constant; instead, they vary periodically with the passage of seasons. Find  $P(t)$  if the population  $P$  satisfies

$$\frac{dP}{dt} = (2 + 3 \cos t)P, \quad P(0) = 100.$$

What is the limit population in the long run?

6 [10 pts] Derive the solution

$$P(t) = \frac{MP_0}{P_0 + (M - P_0)e^{kMt}}$$

of the extinction-explosion initial value problem  $P' = kP(P - M)$ ,  $P(0) = P_0$ .