Final Test **Name:**________________________

MA 238-03 Dec. 12, 2005

Notes: Write your name on answer sheets and show all your work.

1 [10 pts]. Find the solution to the initial value problem:

$$x\frac{dy}{dx} = y^2 + 3x^3y^2, \qquad y(1) = 1.$$

2 [10 pts]. Find a general solution to the equation

$$xy' + 2y = e^{2x}.$$

3 [5 pts]. Use appropriate substitutions to transform the following equations into separable or linear equations without solving the resulting equations. You only need to do one of them.

$$(i) \quad x\frac{dy}{dx} = y + \sqrt{x^2 + y^2}, \qquad (ii) \quad x\frac{dy}{dx} = y + 3x^2y^3.$$

4 [10 pts]. Use the Euler's method with the step size $h = 0.5$ to find an approximation to the solution of

$$y' = x^2 + 2y, \qquad y(1) = 1$$

at $x = 2.5$.

5 [10 pts]. Find general solutions of the following equations:

$$(i) \qquad y'' - 10y' + 25y = 0, \quad (ii) \quad y'' - 10y' + 29y = 0.$$

6 [10 pts]. Use the method of undetermined coefficients to find a particular solution to

$$y'' + 2y' - 3y = xe^{3x}.$$

7 [10 pts]. Use the method of variation of parameters to find a particular solution of

$$y'' - 2y' - 8y = e^{-x} + e^{-2x}.$$

8 [10 pts]. Use the Laplace transforms to solve the initial value problem:

$$x'' - 5x' + 6x = 0, \qquad y(0) = 1, \quad y'(0) = 2.$$

9 [10 pts]. Use the Laplace transforms to solve the initial value problem:

$$x'' + 5x' + 6x = e^t, \qquad x(0) = x'(0) = 0.$$

Do one of the following [15 pts] (if you do both correctly, you will get extra 10 points):

10. (a) Find $L\{f(t)\}$, where $f(t) = \begin{cases} t, & 0 \le t < \pi, \\ 1, & t \ge \pi. \end{cases}$

(b) Solve the initial value problem:

$$x'' + 4x = f(t), \qquad x(0) = x'(0) = 0,$$

where f is given in (a).

11. Solve the initial value problem:

$$x'' + 4x = t * \sin 3t, \qquad x(0) = x'(0) = 0.$$

11 (Bonus). Consider a prolific breed of rabbits whose birth and death rates, β and δ, are each proportional to the rabbit population $P = P(t)$, with $\beta > \delta$. That is, the equation for P is $P' = kP^2$ where $k = \beta - \delta$. Let $P(0) = P_0$. (a) Show that

$$P(t) = \frac{P_0}{1 - kP_0 t}.$$

Note that $P(t) \to \infty$ as $t \to \frac{1}{kP_0}$. This is doomsday.
(b) Suppose that $P_0 = 6$ and that there are nine rabbits after ten months. When does doomsday occur? (Hint: You should find k first.)