

Write your full name in the answer sheet. Show all your work to receive full credit.

1)[10pts]

Classify the PDE

$$2u_{xx} + 21u_{yy} + 12u_{zz} - 12u_{xy} + 12u_{yz} + 3u_y + 1 = 0 .$$

2)[25pts]

Find the solution of the initial boundary value problem

$$\begin{aligned} u_{tt} &= u_{xx}, & 0 < x < \infty, t > 0 \\ u(x, 0) &= \cos(\pi x/2), & 0 \leq x < \infty \\ u_t(x, 0) &= 0, & 0 \leq x < \infty \\ u_x(0, t) &= 0, & t \geq 0 \end{aligned}$$

3)[20pts]

Consider

$$\begin{aligned} X'' + \lambda X &= 0, & -l < x < l \\ X(l) + X(-l) &= 0 \\ X'(l) + X'(-l) &= 0 \end{aligned}$$

Find all eigenvalues and an orthogonal system of eigenfunctions.

4)[25pts]

Solve the boundary value problem for the Laplace equation

$$\begin{aligned} u_{xx} + u_{yy} &= 0, & 0 < x < a, 0 < y < b \\ u(x, 0) = 1, & u(x, b) = 0, & \frac{\partial u}{\partial x}\Big|_{x=0} = 0, \frac{\partial u}{\partial x}\Big|_{x=a} = 0. \end{aligned}$$

5)[20pts]

Find the Fourier-Legendre approximation upto and including the term in $P_3(x)$ of

$$f(x) = x, \quad -1 < x < 1.$$