Write your full name in the answer sheet. Show all your work to receive full credit.

 # 1)[20pts]
The current $i(x,t)$ and voltage $v(x,t)$ in a transmission line obey the equations

$$Li_t + v_x + Ri = 0 \quad \text{and} \quad Cv_t + i_x + Gv = 0\,.$$

Write these equations in the matrix form $\mathbf{U}_t + A\mathbf{U}_x = 0$, to determine their Riemann invariants of i, v for the case of a lossless line in which $R = G = 0$. Show that when the line is distortionless, so $R/L = G/C$, the Riemann invariants can be integrated to give the solutions i and v. Find the expressions for these i and v.

 # 2[20pts]
(a) Use the Rayleigh-Ritz method to find the approximate solution to the Possion's equation $u_{xx} + u_{yy} = -\rho$ in the square $|x| < a$, $|y| < a$ with the boundary condition $u = 0$, where ρ is a constant. Use the trial solution $u(x,t) = c(x^2 - a^2)(y^2 - a^2)$.

 # 3)[20pts]
Use the Galerkin approximation to find the approximate solution to the Poisson's equation $u_{xx} + u_{yy} = -\rho$ in the rectangle $|x| < a$, $|y| < b$ with the boundary condition $u = 0$ on all four sides and the trial solutions $u(x,y) = A_{00}(a^2 - x^2)(b^2 - y^2)$.

 # 4)[20pts]
Use the Green's function method to find the solution to the problem $\Delta u = 0\,, \quad x > 0, \;\; y > 0\,; u(0,y) = 0\,, \;\; y \geq 0\,; \; u_y(x,0) = f(x)\,, \;\; x \geq 0\,.$

 # 5)[20pts]
Consider the scheme

$$\frac{u_j^{n+1} - 2u_j^n + u_j^{n-1}}{\Delta t^2} = c^2 \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2}$$

for approximating the wave equation $u_{tt} = c^2 u_{xx}$. What is the stability condition for this scheme in terms of $s = c^2(\Delta t)^2/(\Delta x)^2$.