MA526 **Fall 2006** **Final Exam**

Write your full name in the answer sheet. Show all your work to receive full credit.

 # 1)[15pts]
Find the solution to the following Cauchy problem for all $t > 0$

$$u_t + u\, u_x = 0\,, \quad x \in \mathbb{R}\,, t > 0\,,$$

$$\text{I.C.:} \quad u(x,0) = \begin{cases} -1 & x \le -1, \\ x & -1 < x < 1, \\ 1 & x \ge 1 \end{cases}$$

 # 2[15pts]
Find the solution of the initial boundary value problem

$$u_{tt} = u_{xx}\,, \quad 0 < x < L, \ \ t > 0$$

$$\text{I.C.:} \quad u(x,0) = x\,, \quad u_t(x,0) = 0\,, \quad 0 < x < L$$

$$\text{B.C.:} \quad u(0,t) = 1\,, \quad u(L,t) = 3, \ \ t > 0$$

 # 3)[15pts]
Solve

$$u_{tt} = u_{xx}\,, \quad x > 0, \ t > 0$$

$$\text{I.C.:} \quad u(x,0) = \sin(3x), \quad u_t(x,0) = x, \ \ x > 0$$

$$\text{B.C.:} \quad u(0,t) = 1 - e^{-t}\,, \quad t \ge 0\,.$$

[**Hint:** You may use characteristic quadrilateral.]

 # 4)[15pts]
Solve the boundary value problem for the Laplace equation

$$u_{xx} + u_{yy} = 0\,, \quad 0 < x < 1, \ \ 0 < y < 1$$

$$\text{B.C.:} \quad u(x,0) = 0\,, \ \ u(x,b) = 100\,, \ \ u(0,y) = 0 \ \ u(a,y) = 0.$$

 # 5)[15pts]
Find the solution of the initial boundary value problem

$$u_t = u_{xx} + u\,, \quad 0 < x < 1, t > 0$$

$$\text{I.C.:} \quad u(x,0) = \phi(x)\,, \quad 0 \le x < 1$$

$$\text{B.C.:} \quad u(0,t) = 0\,, \quad u(1,t) = 0\,, \quad t > 0$$

 # 6)[10pts]
Classify the PDE and reduce it to the canonical form

$$u_{xx} + 6u_{xy} + 9u_{yy} + 3yu_y = 0\,.$$

 # 7)[15pts]
Find the steady-state temperature distribution in a disk of radius 1 if the the upper half of the circumference is kept at 100^o and the lower half is kept at 0^o. That is:
Solve the Laplace equation

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$

for $u(r,\theta)$ in circle $0 \le r \le 1, \ 0 \le \theta \le 2\pi$, subject to the boundary conditions $u = 100$ on the upper half of the circumference and $u = 0$ on the lower half.