

MATH415 Test2

Write Your Full Name in the Answer Sheet. Show all work

1)[30points]

Given:

x	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
f(x)	440	0	-162	-160	-84	0	50	48	0	-64	-90	0

a)[5pts] Approximate $\int_0^6 f(x)dx$ using Trapezoid rule with $h=2$.

b)[5pts] Approximate $\int_0^6 f(x)dx$ using Trapezoid rule with $h=1$.

c)[5pts] Improve the results in parts a) and b) using Romberg integration.

d)[15pts] Find the Newton interpolation polynomial through $f(-5)$, $f(-2)$, $f(3)$ and $f(4)$.

2[20pts]

a) Perform the LU decomposition with pivoting of the following matrix

$$A = \begin{pmatrix} 9 & 2 \\ 8 & 4 \end{pmatrix}$$

Use this to find the solution of the system $Ax = \mathbf{b}$, where A is the above matrix and $\mathbf{b} = (7, 4)^T$.

3)[15pts]

a) [13 pts] Calculate the natural cubic spline interpolating the data

$$\{(0, 0), (1, 2), (2, 3), (3, 4)\}$$

b)[2pts] State the advantages of cubic spline over other interpolations such as Lagrange/Newton interpolation.

4)[15pts]

Apply Gaussian quadrature with $n=2$ to approximate the integral

$$\int_0^1 \frac{1}{\sqrt{1+x^4}} dx .$$

5)[20pts]

Consider using the trapezoid rule T_n to estimate the integral

$$I = \int_1^3 \ln(x) dx$$

Give both a rigorous error bound for $E_n = I - T_n$ and an asymptotic error estimate $\overline{E}_n = I - T_n$. Using the rigorous error bound, determine how large n should be in order that $|I - T_n| \leq 5 \times 10^{-6}$.