

Instructions: Write your name on the answer sheet provided. Show your work to receive credit.

1) [25pts] Find the inverse Laplace transform of the following

$$(i) F(s) = \frac{s+2}{s^2+4s+5} \qquad (ii) F(s) = \frac{5s-6}{s^3-3s}$$

$$[\text{Answer : } (i)f(t) = \cos(t)e^{-2t}]$$

$$\left[\text{Answer : } (ii)f(t) = 2 - 2\cosh(\sqrt{3}t) + \frac{5}{\sqrt{3}}\sinh(\sqrt{3}t) \right]$$

2) [25pts] Use the **method of variation of parameters** to find the general solution of

$$y'' - 2y' - 8y = 3e^{-2x}.$$

$$\left[\text{Answer : } y = y_c + y_p = (c_1e^{4x} + c_2e^{-2x}) - \frac{1}{12}e^{-2x} - \frac{1}{2}xe^{-2x} \right]$$

3) [25pts] A mass weighing 100lb is attached to the end of a spring that is stretched 1in. by a force of 100lb. A force of $2\cosh(3t)$ is being applied on the mass. If the mass is set in motion with $x(0) = 0$ and $x'(0) = 0$, find $x(t)$ for the resulting motion.

$$\left[\text{Answer : } x(t) = \frac{16}{9825}(\cosh(3t) - \cos(\sqrt{384}t)) \right]$$

4)[25pts] Use Laplace transform to solve the following system of DEs.

$$x' = 2x + y, \quad y' = 6x + 3y$$

$$x(0) = 0, \quad y(0) = 2.$$

$$\left[\text{Answer : } x(t) = \frac{2}{5}(e^{5t} - 1); \quad y(t) = \frac{2}{5}(2 + 3e^{5t}) \right]$$