

Instructions: Write your name on the answer sheet provided. Show all of your work and write your answers on the answer sheet. Please box your answers.

1 [10pts] Solve the initial value problem

$$y'(1+x^2) + xy = 0, \quad y(0) = 1.$$

2 [10pts] Solve the initial value problem

$$y' - 2y = xy^3, \quad y(1) = 2\sqrt{2}.$$

3[10pts] Solve the differential equation

$$(3x^2 + 2y)dx + (2x + 2y)dy = 0.$$

4[10pts] Use Euler's method with step size $\Delta t = 0.5$ to approximate the solution of

$$y' = \frac{1+x}{1+y^2}, \quad y(0) = 1$$

at $t = 1$.

5[10pts] A tank initially contains a solution of 10 pounds of salt in 60 gallons of water. Water with $1/2$ pound of salt per gallon is added to the tank at 6 gallon/minute, and the resulting solution leaves at the same rate. Find the quantity $Q(t)$ of salt in the tank at time t .

6 [10pts] A 96-lb weight is dropped from rest in a medium that exerts a resistive force with magnitude proportional to the speed. Find its velocity as a function of time if its terminal velocity is 128 ft/s .

7[10pts] Use Laplace transform to solve the initial value problem

$$\begin{aligned} x'_1 + x_1 + 2x'_2 &= 0, & x_1(0) &= 0. \\ x'_1 - x'_2 + x_2 &= 0, & x_2(0) &= 1. \end{aligned}$$

8 [10pts] Use the method of variation of parameters to find a particular solution, given the solutions y_1, y_2 of the complementary equation.

$$x^2 y'' - 4xy' + 6y = x^{5/2}, \quad y_1 = x^2, y_2 = x^3.$$

9[10pts] Use Laplace transform to solve the following initial value problem

$$\begin{aligned} y'' - 3y' + 2y &= te^{-t} \\ y(0) &= 0, \quad y'(0) = 0. \end{aligned}$$

10 [10pts] An 8-lb weight stretches a spring 8 inches in equilibrium. It is attached to a dashpot with damping constant $c=0.5$ lb-s/ft and subjected to an external force $f(t) = 4 \cos(2t)$ lb. Find the displacement at any time t .