

MATH201 MODEL EXAMINATION

- 1a) Find the equation of the tangent line to the curve C given by $\mathbf{r}(t) = (\cos(t), \sin(t), t)$ at the point $(1,0,0)$.
- b) Find the length of the curve C from $(1,0,0)$ to $(1,0,2\pi)$.
- c) Find the curvature of C at $(1,0,0)$.
- 2 The length and width of a rectangle are measured as 30cm and 24cm, respectively, with a possible error of 0.2 cm in each dimension. Use differentials to estimate the maximum error in calculating the area of the rectangle.
- 3 The radius of a right circular cylinder is **decreasing** at a rate of 1.2 cm/s while its height is **increasing** at a rate of 3 cm/s. At what rate is the volume of the cylinder is changing when the radius is 80cm and the height is 150cm.
- 4a) If $f(x, y) = x^2 + y$, find the rate of change of f at the point P(2,0) in the direction from P to Q(1/2,2).
- b) In what direction does f change most rapidly? What is this maximum rate of change?
5. Find the dimensions of the rectangular box with maximum volume if the sum of the lengths of its 12 edges is 8.
- 6a) Evaluate $\iiint_E x \, dv$, where E is the solid tetrahedron bounded by the four planes $x=0, y=0, z=0$ and $x + y + z = 1$.
7. Let C be the arc of the parabola $y = x^2$ from $(0,0)$ to $(1,1)$. Evaluate the line integrals
- a) $\int_C x \, ds$, where s is the arc length,
- b) $\int_C x \, dy$.
8. Find the mass and center of mass of a thin wire in the shape of a quarter-circle $x^2 + y^2 = 4, x \geq 0, y \geq 0$, if the density function is $\rho = x + y$.
9. A lamina occupies the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant. Find its center of mass if the density at any point is proportional to its distance from the x-axis.
- 10a) Show that $\mathbf{F} = 2xi + 2yj$ is conservative.
- b) Find a function f such that $\mathbf{F} = \nabla f$.
- c) Use Fundamental Theorem of line integral to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is any curve from $(0,0)$ to $(2,1)$.
11. Use Greens Theorem to evaluate $\int_C x^2 dx + xy dy$, where C is the triangular curve connecting the points $(0,0), (1,0)$ and $(0,1)$.
12. Find the work done by a force of 20lb acting in the direction $N50^{\circ}W$ (50° measured from North) in moving an object 4 ft due west.
13. If $R = [-2, 2] \times [-1, 1]$, use the Midpoint Rule with $m = n = 2$ to estimate the value of
$$\int \int_R (y + x^2 y) dA.$$
14. A gun has muzzle speed 120m/s. What angle of elevation should be used to hit an object 500 m away.